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Finite strain quadrilateral shell using least-squares fit of relative Lagrangian in-plane strains ☆

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ABSTRACT

This work presents a finite strain quadrilateral element with least-squares assumed in-plane shear strains (in covariant/contravariant coordinates) and classical transverse shear assumed strains. It is an alternative to enhanced-assumed-strain (EAS) formulation and, in contrast to this, produces an element satisfying ab initio the Patch-test. No additional degrees-of-freedom are present, unlike EAS. Least-squares fit allows the derivation of invariant finite strain elements which are both in-plane and out-of-plane shear-locking free and amenable to standardization in commercial codes. With that goal, we use automatically generated code produced by AceGen and Mathematica to obtain novel finite element formulations. The corresponding exact linearization of the internal forces was, until recently, a insurmountable task. We use the tangent modulus in the least-squares fit to ensure that stress modes are obtained from a five-parameter strain fitting. This reproduces exactly the *in-plane* bending modes. The discrete equations are obtained by establishing a four-field variational principle (a direct extension of the Hu–Washizu variational principle). The main achieved goal is coarse-mesh accuracy for distorted meshes, which is adequate for being used in crack propagation problems. In addition, as an alternative to spherical interpolation, a consistent director normalization is performed. Metric components are fully deduced and exact linearization of the shell element is performed. Full linear and nonlinear assessment of the element is performed, showing similar performance to more costly approaches, often on-par with the best available shell elements.

1. Introduction

Finite strain plasticity and fracture simulations with finite elements (cf. [8,10]) are peculiarly demanding with respect to numerical efficiency, Newton iteration robustness and mesh distortion insensitivity. This is relevant in the edge-based algorithms recently proposed [12] when applied to quadrilaterals. Many of the intricate element formulations, such as enhanced-assumed-strain, hybrid stress, discrete Kirchhoff (DK, cf. [14]), are suitable for smooth problems where the mesh distortion sensitivity is not a crucial factor and governing equations do not contain discontinuities. In addition, costs associated with convergence difficulties and static condensation (specifically with EAS) can also be high. We take a different approach here: starting with a mixed 4-field functional (displacement field, director field, components of the local Cauchy–Green tensor and the corresponding stress-like Lagrange multipliers), we discretize the resulting Euler–Lagrange equations making use of suitable shape functions. A complete testing program is then performed. The set of obstacle problems for shells are the classical plate and shell benchmarks and extensions to finite strains. Testing elements in finite strains is also important since some instabilities have been found in the past (see [22] for a report with the Morley-based shell). Element technology for quadrilaterals is too vast to be accounted in a single article and many elements proposed in the last three decades vary only slightly in performance for the same number of degrees-of-freedom. Some important works must be mentioned. A milestone in the removal of transverse shear locking was achieved with the assumed natural strain (ANS) technique in 1984 and 1986 [24,36]. A decade earlier,

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